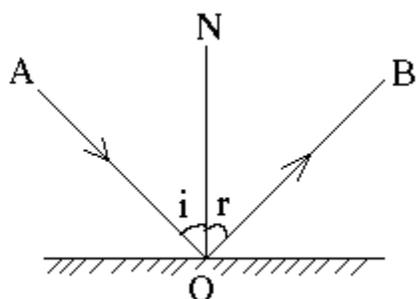


# UNIT-IX RAY OPTICS

Light is a form of energy. The ray treatment of light forms RAY OPTICS.

## Reflection



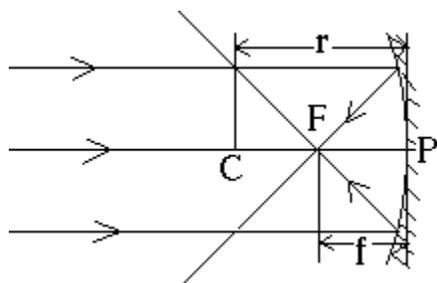
Angle of incidence 'i' is the angle between incident ray AO and normal ON at the point of incidence. Angle of reflection 'r' is the angle between the reflected ray OB and normal ON.

## Laws of reflection.

- 1) The angle of incidence is equal to the angle of reflection.
- 2) The incident ray, the reflected ray and normal lies in the same plane.

## Spherical mirrors.

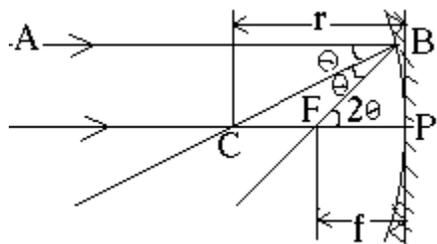
A spherical mirror is a reflecting surface, which forms the part of a sphere. When reflection takes place from inner surface of a mirror it is called concave mirror and reflection takes place from outer surface, the mirror is called convex mirror.



P-Pole, C-Centre of curvature, r- Radius of curvature,  
CP- Principle axis- Line, F-Principle focus, f-focal length.

- 1) Pole:- Pole of a spherical mirror is the geometrical centre of the mirror.
- 2) Centre of curvature:- of a spherical mirror is the centre of a sphere of which mirror forms a part.
- 3) Radius of curvature:- of a spherical mirror is the radius of a sphere of which the mirror forms a part.
- 4) Principle axis:- of a spherical mirror is the straight line passing through the centre of curvature and pole of the mirror.
- 5) Principle focus:- A narrow parallel beam of light parallel and close to the principle axis is incident on a spherical mirror, after reflection the beam converges to a point on the principle axis in the case of concave mirror or appears to diverge from a point on the principle axis in the case of convex mirror. This point is called principle focus of a spherical mirror.
- 6) Focal length:- is the distance between the pole and principle focus of the mirror.

## Relation between radius of curvature and focal length (f).



From the figure,  $\theta = \frac{BP}{CP} = \frac{BP}{r}$ -----(1).

Since AB is close to the principle axis,  
 $2\theta = \frac{BP}{FP} = \frac{BP}{f}$ -----(2).

Eqn--> (1)/(2) gives  $\frac{1}{2} = f/r$ . Or  **$r = 2f$**

Formation of image by spherical mirrors..

Geometrical construction.

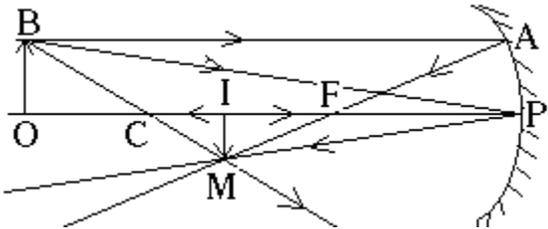
- 1) A ray parallel to the principle axis after reflection passes through the principle focus of a concave mirror or appear to diverge from the principle focus in the case of concave mirror.
- 2) A ray incident normally on the spherical mirror after reflection retraces the path.
- 3) A ray passing through the principle focus of a concave mirror after reflection travels parallel to the principle axis.
- 4) A ray incident at any point on the spherical mirror will be reflected according to the laws of reflection.

Sign convention.

- 1) All distances are measured from the pole of the mirror.
- 2) Distances measured in the direction of incident light are taken as +ive and opposite direction are taken as -ive.
- 3) The heights measured upwards and perpendicular to the principle axis are taken as +ive and vice versa.

Mirror equation :- Concave mirror forming real image.

Formation of image of an object according to the laws of geometry of ray optics is shown below.



Let OB is an object and IM is the image. According to sign convention V, U and 'f' are taken positive.

From triangles IMF and APF,  $\frac{IM}{AP} = \frac{IF}{FP} = \frac{(V-f)}{f}$ ----- (1)

{since PI=V and PF=f} From triangles OBP and IMP,  $\frac{IM}{OB} = \frac{PI}{PO} = \frac{V}{U}$  --(2) since PO=U}.

From eqn (1) and (2),  $\frac{IM}{AP} = \frac{IM}{OB}$ . {since OB = AP}

$\therefore \frac{V}{U} = \frac{(V-f)}{f}$  -----(3). Or  $\frac{V}{U} = \frac{V}{f} - 1$  -----(4).

Dividing eqn (4) by V, we have  $1/U = 1/f - 1/V$ .

Or  $\boxed{1/U + 1/V = 1/f}$ . This formula is valid for convex mirror also (applying proper sign convention).

Magnification.

It is the ratio of size of the image to the size of the objects.

ie linear magnification = image height/object height.

Magnification in terms of U and V:- magnification,  $\boxed{m = V/U}$ .

Magnification in terms of U and f :-

By law of distances,  $1/U + 1/V = 1/f$ . -----(1)

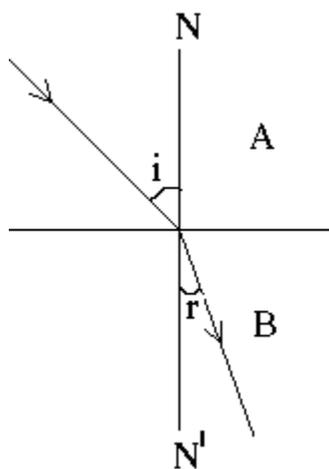
Substituting the value, V = mU in eqn (1), we have  $1/U + 1/mU = 1/f$ .

Or  $\frac{1}{mU} + \frac{1}{U} = \frac{1}{f}$ . ie  $\frac{1}{m} + \frac{1}{U} = \frac{1}{f}$ . Or  $\boxed{m = f/(U-f)}$ .

Magnification in terms of V and f :-

$\boxed{m = (V-f)/f}$

Refraction at plane surface.



Bending of light ray when it passes from one transparent medium into another, at the surface of separation, is called refraction.

If the ray enters to a denser medium from a rarer medium, ray bends towards the normal. If from denser to rarer the ray bends away from the normal after refraction.

Laws of refraction.

- 1) The incident ray, the refracted ray and the normal to the surface of separation lies in the same plane.
- 2) The ratio of the sin of angle of incidence to the sin of angle of refraction is constant for a given pair of medium and given colour of light. This is called Snell's law.

If 'i' is the angle of incidence and 'r' the angle of refraction then,  $\sin i / \sin r = \text{constant}$ . This constant is called the refractive index of the

second medium B with respect to the first medium A and is represented by,  ${}_A\mu_B = \sin i / \sin r$

Absolute refractive index of a medium.

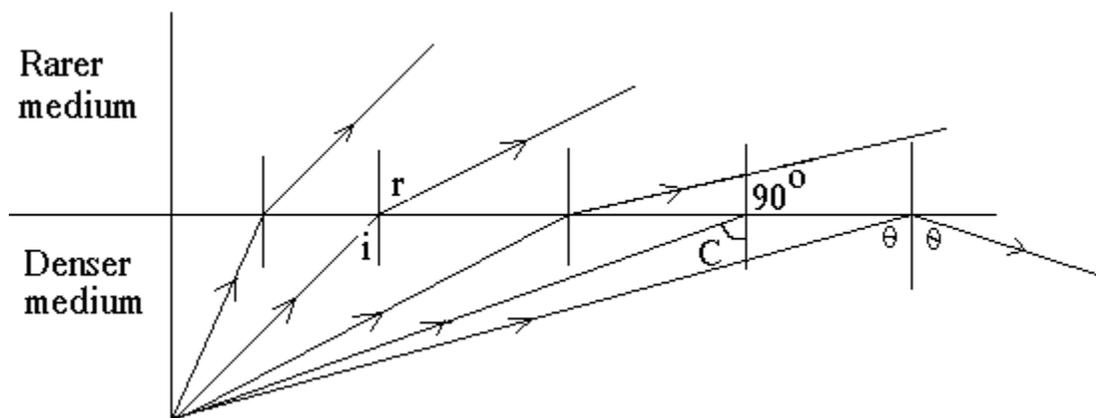
If ray travels from air (vacuum) into a medium, its refractive index is known as absolute refractive index.

$\mu = \sin i / \sin r$

Since the velocity changes in different media,

Refractive index of medium = velocity of light in vacuum / velocity of light in medium.

Critical angle and total internal reflection.



As the angle of incidence increases the angle of refraction also increases, when light travels from a denser medium into air, as shown above. The angle of incidence in a denser medium for which the angle of refraction in air is  $90^\circ$  is called critical angle of the medium.

Refractive index of medium =  $\frac{\sin 90^\circ}{\sin C}$  =  $\frac{1}{\sin C}$ , where C is critical angle.

ie  $\mu = \frac{\sin 90^\circ}{\sin C} = \frac{1}{\sin C}$ . Or  $\mu = \frac{1}{\sin C}$ , where C is critical angle.

Total internal reflection.

When light travels from denser medium into a rarer medium and angle of incidence is greater than the critical angle of the denser medium, the entire light is reflected back into the medium. This phenomenon is called

total internal reflection.

Conditions for total internal reflection.

- 1) Light must travel from a denser medium into a rarer medium.
- 2) The angle of incidence must be greater than the critical angle.

Applications of Total internal reflection

1. Total reflecting prisms, 2. Mirage, 3.Brilliance of diamond (Refractive index = 2.42, hence critical angle is low =  $24.4^\circ$ )

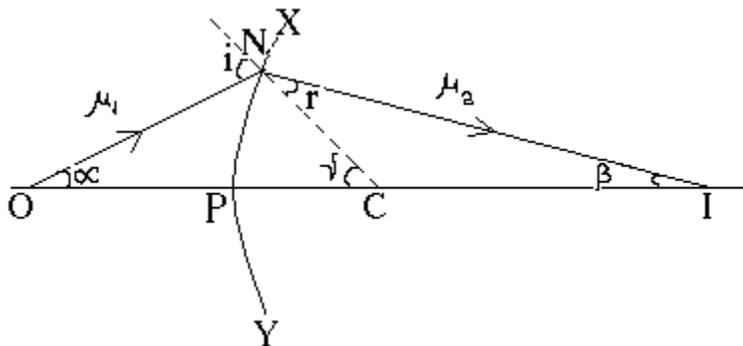
4. Optical fibres.

The principle of total internal reflection is used in the construction of optical fibres. It consists of thousands of very fine glass/quartz fibres. The fibres can be 0.0001cm in diameter and refractive index 1.7. These are coated with a thin layer of material of low refractive index 1.5. The light entering the fibre at a small angle to the axis undergoes repeated total internal reflection along the fibre and finally emerges.

Uses.

Optical fibres can be used as ‘Light pipes’ in optical and medical examination. Used for optical signal transmission, transmission and reception of electrical signals. Optical fibres are widely used in all types of communication purposes.

Refraction at spherical surfaces.



XY is a convex surface separating two transparent media having refracting indices  $\mu_1$  and  $\mu_2$ . O is an object placed at a distance u from the pole P of the spherical surface. A ray ON incident at an angle ‘i’ is refracted at an angle ‘r’ along NI, such that  $\frac{\sin i}{\mu_1} = \frac{\sin r}{\mu_2}$  ie  $\mu_1 \sin i = \mu_2 \sin r$ -----(1)

I is the image of O, and is formed at a distance ‘v’ from p.

Let  $\angle NOP = \alpha$ ,  $\angle NIP = \beta$ ,  $\angle NCP = \gamma$  and  $PC = R$  – radius of curvature of the spherical surface.

From the  $\Delta ONC$ ,  $i = \alpha + \gamma$ ------(2). From the  $\Delta NIC$ ,  $r = \gamma - \beta$ ------(3).

For small aperture,  $\sin i = i$ , and  $\sin r = r$ .

$\therefore \mu_1 \times i = \mu_2 \times r$ . ie  $\mu_1(\alpha + \gamma) = \mu_2(\gamma - \beta)$ . Or  $\mu_1 \alpha + \mu_1 \gamma = \mu_2 \gamma - \mu_2 \beta$ .

Or  $\mu_1 \alpha + \mu_2 \beta = (\mu_2 - \mu_1)\gamma$ . But  $\alpha = \frac{PN}{u}$ ,  $\beta = \frac{PN}{v}$ ,  $\gamma = \frac{PN}{R}$ .

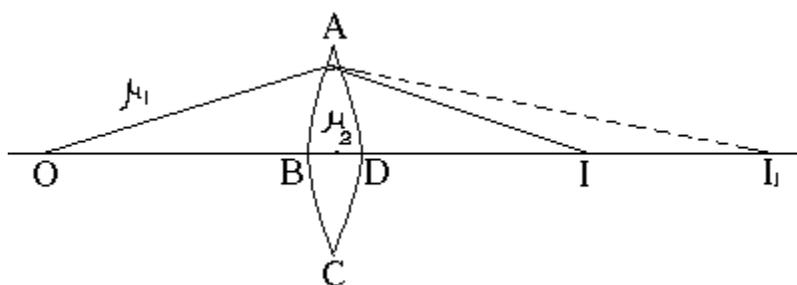
$\mu_1 \left(\frac{PN}{u}\right) + \mu_2 \left(\frac{PN}{v}\right) = (\mu_2 - \mu_1) \frac{PN}{R}$ .

Or  $\frac{\mu_1}{u} - \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R}$ . Applying sign, u is -ive, v is +ive and R is +ive.

Hence  $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}}$ .

**Refraction by a lens**

Distance law for thin lens.



Consider a thin lens bounded by two spherical surfaces ABC and ADC. Let  $r_1$  and  $r_2$  be the radii of curvatures of the two faces respectively. Let  $\mu_2$  be the refractive index of the material of the lens and  $\mu_1$  that of the medium in which the

lens is placed. O is an object at a distance 'u' from B. After refraction at the face ABC, an image would have formed at I<sub>1</sub>, distant v<sub>1</sub> from B,

$$\text{such that } \frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{r_1} \text{-----(1)}$$

This image acts as a virtual object for the refraction at the face ADC (r<sub>2</sub> is -ive). The final image is formed at I, distant v from D. Here ray passes through the lens (μ<sub>2</sub>) and enters the rarer medium (μ<sub>1</sub>) and object is virtual (object distance u = -v<sub>1</sub>).

$$\text{So the equation becomes, } \frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{(\mu_2 - \mu_1)}{r_2} \text{----- (2).}$$

$$\text{Adding eqn (1) and (2) we have } \frac{\mu_1}{v} + \frac{\mu_1}{u} = (\mu_2 - \mu_1) \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \text{-----(3).}$$

$$\text{Dividing through out by } \mu_1, \frac{1}{v} + \frac{1}{u} = \frac{(\mu_2 - 1)}{\mu_1} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \text{-----(4).}$$

For an object at infinity the image is formed at the focus, ie u = ∞ and v = f, hence

$$\frac{1}{f} = \frac{(\mu_2 - 1)}{\mu_1} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \text{-----(5).}$$

If μ is the refractive index of the lens and it is placed in air, then, μ<sub>2</sub> = μ and μ<sub>1</sub> = 1,

$$\therefore \frac{1}{f} = (\mu - 1) \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \text{-----(6). This is the famous lens makers formula.}$$

From eqns (4) and (6),  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ . This is the law of distances for lens.

Magnification

. Magnification of a lens is the ratio of the size of the image to the size of object.

Linear magnification = Height of image/Height of object. = h<sub>i</sub>/h<sub>o</sub> = v/u.

Arial magnification = Area of image / Area of object = v<sup>2</sup>/u<sup>2</sup>.

Power of a lens. [P]

Power of a lens is the reciprocal of the focal length in meters.  $P = 1/f$ . Unit of power is **Dioptre**.

Conventionally power of converging lens is taken as positive and that of diverging lens is negative.

Combination of thin lenses in contact.

Consider two thin lenses L<sub>1</sub> and L<sub>2</sub> having focal lengths f<sub>1</sub> and f<sub>2</sub> respectively. Let the two lenses combine together as shown in the figure. Let 'O' be the object (object distance 'u') and I<sub>1</sub> be the image formed at v<sub>1</sub> distance when L<sub>2</sub> is absent. Then 1/v<sub>1</sub> - 1/u = 1/f<sub>1</sub>-----(1)

The image I<sub>1</sub> acts as virtual object for the lens L<sub>2</sub> and forms the final image at I, v distant from the lens.

$$\text{Then } 1/v - 1/v_1 = 1/f_2 \text{-----(2)}$$

$$\text{Adding eqn (1) and (2) we have } 1/v - 1/u = 1/f_1 + 1/f_2 \text{-----(3)}$$

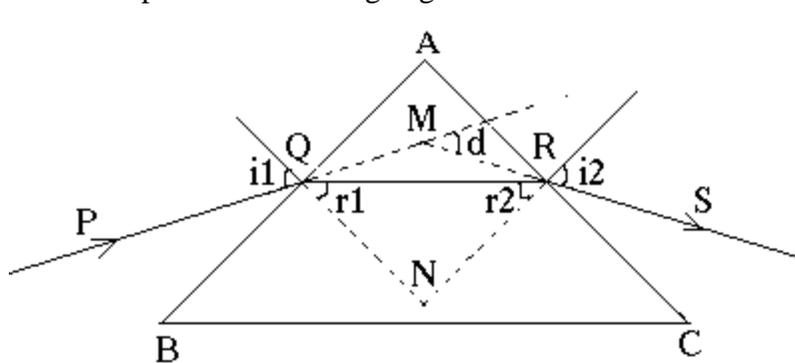
If 'F' is the effective focal length of the combination, and 'u' and 'v' are the object and final image distance respectively, then 1/v - 1/u = 1/F -----(4)

Comparing eqn (3) and (4) we have  $1/v - 1/u = 1/F$

Power of the combination is  $P = P_1 + P_2$

Refraction through prism.

ABC is a prism of refracting angle A. AB and AC are faces of the prism and BC is the base.



A ray PQ incident on the face AB at an angle 'i', is refracted along QR at an angle 'r', such that the refractive index 'μ' of the material is given by,

$$\mu = \frac{\sin i}{\sin r} \text{-----(1).}$$

Refracted ray QR incident on the face AC at an angle  $r_2$  and emerges out at an angle  $i_2$ . The angle between the directions of the incident ray and emergent ray is called angle of deviation 'd'.

Let the two normal at Q and R meet at N.

In the quadrilateral AQNR,  $\angle AQN = \angle ARN = 90^\circ$ .  $\therefore \angle A + \angle N = 180^\circ$ .

In the  $\Delta QNR$ ,  $r_1 + r_2 + \angle N = 180^\circ$ .

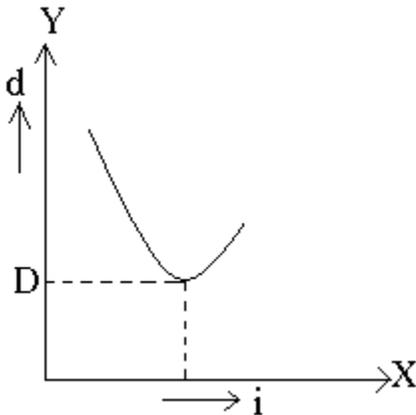
$\therefore$  Angle of prism,  $\angle A = r_1 + r_2$ -----(2).

From  $\Delta QMR$ , the deviation,  $d = \angle MQR + \angle MRQ$

$$= (i_1 - r_1) + (i_2 - r_2)$$

$$= i_1 + i_2 - (r_1 + r_2).$$

Or angle of deviation,  $d = i_1 + i_2 - A$ -----(3).



For a given prism, as the angle of incidence increases, the angle of deviation decreases and reaches a minimum value and then increases as shown in the graph. The smallest value of deviation is called angle of minimum deviation 'D'. The ray undergoing minimum deviation passes symmetrically through the prism, and hence,

$$d = D, \quad i_1 = i_2 = i, \quad r_1 = r_2 = r.$$

$$\text{From eqn (2), } A = 2r \text{ or } r = A/2 \text{-----(4)}$$

$$\text{From eqn (3), } D = 2i - A. \text{ Or } 2i = A + D.$$

$$\text{Or } i = (A + D)/2 \text{----- (5).}$$

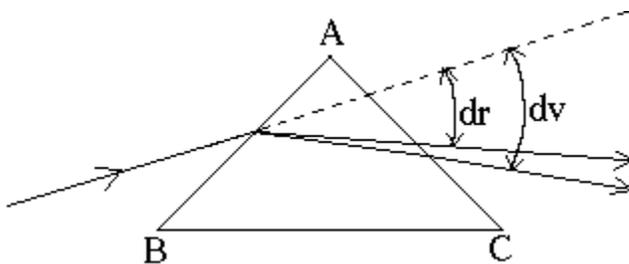
Substituting eqn (4) and (5) in eqn (1) we have,

$$\mu = \frac{\sin(A + D)/2}{\sin(A/2)}. \text{ This is known as prism formula.}$$

### Dispersion of light.

When a narrow beam of white light is passed through a prism it is not only deviated but also split up into different colours. This display of colours is known as spectrum of the source of light.

The splitting of composite beam of light into its constituent colours is known as Dispersion.

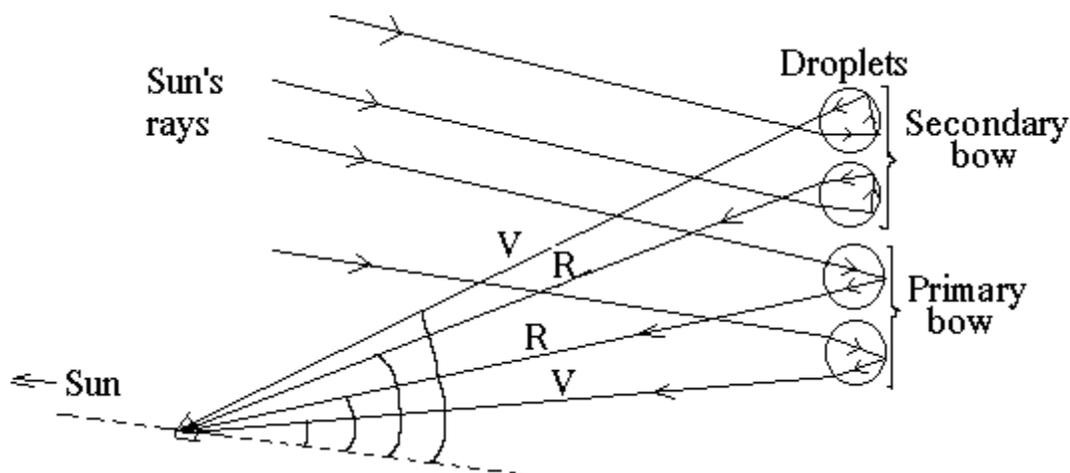


The deviation produced by a prism is different for different colours. Since deviation depends on the refractive index of the prism, refractive index is different for different colours.

### Some Natural Phenomenon Due To Sunlight

#### The Rainbow

Rainbow is an example of dispersion of sunlight by water drops in the atmosphere. To have rainbow, the sun be shining in one part of the sky and the rain be falling in the opposite part of the sky.



A ray of light through drop undergoes dispersion with violet deviated most and red light least. Due to reflection and refraction of the dispersed light the primary rainbow has the red colour on the top and violet at the bottom.

### Scattering of light.

Light is scattered by small dust particles and molecules of air. Shorter wavelength waves are scattered more readily than longer waves. The intensity of scattered light is inversely proportional to the fourth power of the wavelength of the incident light. i.e.  $I \propto 1/\lambda^4$ . This is called Rayleigh's law of scattering.

### Blue colour of sky.

The blue of the sky is due to the scattering of light by small particles of the atmosphere. Shortest wavelength light (violet, indigo) is scattered and removed on the way to observer's eye. The resultant light that reaches the observer has maximum intensity of blue region. At sun set or sun rise, sun light has to travel through long dust path and practically all the violet and blue have been scattered out, hence remaining colour is red.

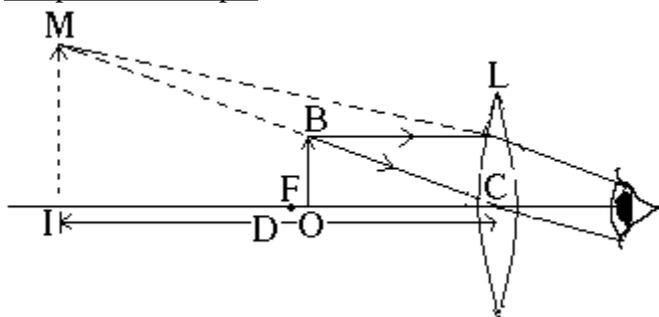
## OPTICAL INSTRUMENTS

### Distance of distinct vision (DDV).

The least distance from eye at which an object could be seen clearly without strain of eye is known as least distance of distinct vision. The DDV for normal eye is 25cm.

### The Eye

#### Simple microscope.



A simple microscope is a convex lens of suitable focal length. The object to be magnified is placed within the focus and the image is adjusted to be formed at the DDV.

Magnifying power (M) or angular magnification.

It is the ratio of the angle subtended at the lens by the image to the angle subtended at the lens

by the object.

$$\text{ie } M = \frac{\text{angle subtended at the lens by the image}}{\text{angle subtended at the lens by the object}} \quad \text{ie } M = \frac{IM/D}{OB/D} = \frac{IM}{OB}$$

. This is same as linear magnification. Or  $M = V/U$ .

From law of distances,  $1/V - 1/U = 1/f$ , multiplying throughout by V, we have  $1 + V/U = V/f$ . Applying sign convention  $U = +ve$ ,  $V = -ive$  and  $f = +ive$ ,

ie  $1 - V/U = -V/f$ . Or  $1 - M = -D/f$  [ $V = D$ ]

$\therefore \boxed{M = 1 + D/f}$ .

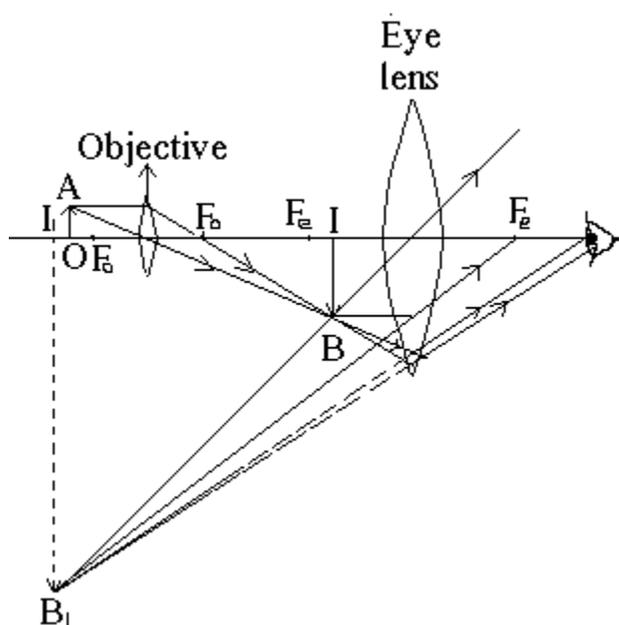
A magnifier frequently places the object at the focus so that parallel rays enter the eye and the image is formed at infinity rather than 25cm.  $\therefore$  Magnifying power,  $M = \frac{(OB/f)}{(OB/D)} = \frac{D}{f}$ .

Compound microscope.

It consists of two converging lenses, one called objective (short focal length) and the other called eyepiece (longer focal length). The two lenses are fitted at the ends of a tube and the distance between them is adjustable.

Working.

The object is placed slightly beyond the focus of the objective, which forms an enlarged real image. This image is formed within the focus of the eye-piece and an enlarged virtual image is obtained. The final image is adjusted to be at the least DDV.



Magnification.

Magnification produced by the compound microscope = Size of image / size of object =  $IM/OB$

Multiplying numerator and denominator by  $I'M'$ , We have  $\frac{IM \times I'M'}{I'M' \times OB} = M_e \times M_o$

I.e. The product of magnification produced by the eyepiece and the magnification produced by the objective gives magnification produced by a compound microscope.

$M = M_o \times M_e$ , where  $M_o$ - magnification by objective and  $M_e$ - magnification by eye piece.

$M = \frac{V_o}{U_o} \times [1 + \frac{D}{f_e}]$ ,

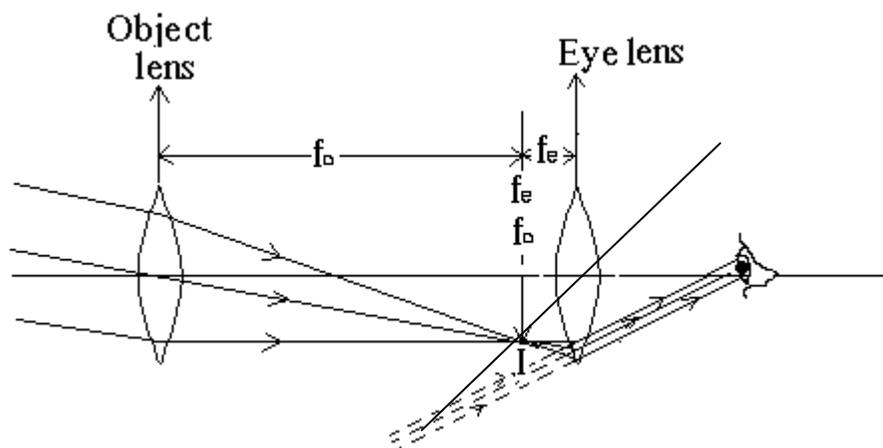
where  $V_o$  and  $U_o$  are image and object distance from objective, and  $f_e$ -focal length of eyepiece.

Another expression for  $M = \frac{L}{f_o} \times \frac{D}{f_e}$ ,

where  $f_o$  and  $f_e$  are the focal lengths of objective and eyepiece respectively.  $L$  – the distance between objective and eyepiece.

Refracting telescope.

It is an instrument used for observing distant objects. It consists of an objective of large focal length and aperture, and eyepiece of short focal length. They are fitted at the ends of a tube and the distance between them is adjustable.



### Working.

Light from a very distant object forms a real image at the focal plane of the objective. This image is adjusted to be at the focal plane of the eyepiece so that the final image is formed at infinity. When the final image is at infinity the telescope is said to be in normal position.

### Magnifying power of a telescope.

It is the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye.

$M = \text{Angle subtended by the image at eye} / \text{Angle subtended by the object at eye}$   
 $= \text{Angle subtended by the image at eye-piece} / \text{Angle subtended by the object at objective.}$

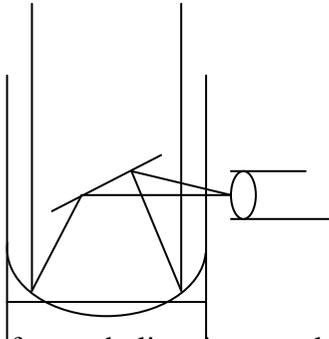
$$M = \beta = \frac{(IM/IC)}{\alpha} = \frac{CI}{C'I} = \frac{f_o}{f_e} \quad \text{i.e.} \quad \boxed{M = f_o/f_e}.$$

### Terrestrial telescope.

In this case, an erecting lens (convex lens) is placed in between objective and eyepiece of astronomical telescope. The inverted image formed by the objective is at 2F of the erecting lens. The final image adjusted to be formed at the least distance of distinct vision by the eyepiece.

Magnification power when the final image formed at DDV,  $M = \frac{f_o}{f_e} [1 + \frac{f_e}{D}]$ .

### Reflecting Telescope:-



It consists of a parabolic mirror made of an alloy of copper and tin. It is fitted at one end of a metal tube; the other end of it is open. Rays from distant object allowed to fall on the mirror. After reflection, before converging the rays to a point are allowed to fall on a plane mirror inclined  $45^\circ$  to the axis of the parabolic mirror. An eyepiece is fitted at right angles to the tube to receive the rays from plane mirror as shown in the figure.

### Resolving power:-

The ability of an optical instrument to form distinctly separate images of two objects very close together is called resolving power.

### Microscope.

Limit of resolution of a microscope is the least distance between two points of an object which can be seen clearly. (d)

The limit of resolution of a microscope,  $\boxed{d = \lambda/2n \sin\theta}$ , where  $\lambda$ -wave length of light used, n-refractive index of the medium between object and objective and ' $\theta$ ' the half angle of the cone of light from the object. ' $n \sin\theta$ ' is called numerical aperture of the microscope.

The reciprocal of limit of resolution is called 'resolving power'.

$$\text{I.e Resolving power} = \frac{1}{d} = \frac{2n \sin\theta}{\lambda}$$

### Telescope:-

Limit of resolution of a telescope is the smallest angular separation between two objects, which can be seen through the telescope.

I.e the limit of resolution,  $\boxed{d\theta = 1.22\lambda/D}$ , where 'D' is the diameter of the telescope objective and ' $\lambda$ ' is the wavelength of the light from the objects.

$$\text{I.e Resolving power of telescope} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$